

ANALOGOUS FUNCTION THEORIES FOR CERTAIN SINGULAR PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. Transmutation operators are used to establish analogous function theories for the heat equation, the equation of generalized axially symmetric potential theory, and the Euler-Poisson-Darboux equation. Under these transformations correspondences are established relating fundamental solutions, polynomial solutions, associated functions, generating functions, and expansion theorems including Fourier transform criteria. In some cases, the transmutation operators must be interpreted in the generalized sense as acting on distributions.

1. Introduction. In [14], D.V. Widder pointed out numerous analogies between classical function theory and expansion theory for solutions of the heat equation. He did this by comparing, by means of a table, the underlying function sets, the generating functions for these sets, the orthogonality properties of these sets, etc. More recently E.G. Dunne and D.G. Mugler [7] extended these analogies by examining the corresponding functions and generalized function sets for the one-dimensional wave equation. Again, comparisons were made by means of a table. In [5], the authors made use of integral transformations connecting the solutions of the heat equation with solutions of the Laplace and wave equation to relate isomorphically various aspects of these function theories as exhibited in these tables. When taken in the classical sense, these integral transformations impose restrictions on the growth of the functions to be transformed. However, when interpreted in the generalized sense they permit almost complete reconciliation with the tables of [7] and [14].

The purpose of this paper is to extend the methods and results of [5] to function theories corresponding to the equation of generalized axially symmetric potential theory (GASPT) and the Euler-Poisson-Darboux

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