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NEW USER-TRANSPARENT EDGE CONDITIONS FOR BICUBIC SPLINE SURFACE FITTING

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0. Introduction. Surface fitting is an important element in many problems in applied mathematics. In particular, in aircraft engine design and development, the accurate representation of geometric part data for use in engineering analysis programs or for determination of appropriate processing operations is a key requirement in computer-aided design and computer-aided manufacturing (CAD/CAM) applications. This paper is concerned with the development of practical techniques for surface representation.

1. The problem. We consider the problem of producing a bicubic spline surface fit which interpolates to given function data on a rectangular grid. This problem, the two-dimensional analog of univariate cubic spline curve fitting, arises in a very broad spectrum of engineering applications, such as the mathematical modeling of airfoils in aircraft engine design.

2. Background: the one-dimensional case. Because our solution to the problem is a natural generalization of the one-dimensional case, we begin with a brief review of our work on the univariate problem. A more rigorous and complete discussion can be found in Ahlberg, Nilson, and Walsh [1] or deBoor [2]; our focus here is primarily on practical procedures.

In [1], the cubic spline fit is presented as the mathematical analog of the draftsman's spline, and the need to prescribe additional boundary conditions (i.e., other than interpolation) in order to determine this fit is introduced as a natural consequence of the underlying physical model. In [4], using a natural or "cardinal" set of basis functions for the case of equally spaced data on an infinite grid, Nilson showed constructively that, for a set of n equally spaced data points, the restrictions of $n + 2$ of these basis functions to the data interval could be used to develop a two-parameter family of cubic spline interpolants to the given data; additional constraints, the so-called "end" conditions, are necessary to determine a unique interpolating spline. Specification of end slopes, or

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