ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 2, Spring 1984

MORSE FUNCTIONS AND SUBMANIFOLDS OF HYPERBOLIC SPACE

TOSHIHIKO IKAWA

ABSTRACT. We study the hypersurface of the Hyperbolic space H^{2m+1}. In H^{2m+1}, there are Morse functions. If we assume that these Morse functions have index 0, m or $2m$ at all these critical points, then we can determine the hypersurface.

1. Introduction. Let M be a differentiable manifold of class C^∞. By a Morse function f of M, we mean a differentiable function on M having only non-degenerate critical points.

In [5], Nomizu and Rodriguez showed the following result of a geometric nature analogous to Reeb's Theorem. If M ($\dim M = n \geqq 2$) is a connected, complete Riemannian manifold isometrically immersed in R^{n+p} such that every Morse function of the form L_p has index 0 or n at any of its critical points, then M is embedded as a Euclidean subspace or a Euclidean n-sphere. Here $L_p(x) = (d(x, p))^2$, $p \in R^{n+p}$, $x \in M$ and d is the Euclidean distance function (see also [4]).

Cecil [1] characterized the metric spheres in hyperbolic space H^m in terms of hyperbolic distance functions L_p. In [2], Cecil and Ryan studied umbilic submanifolds in a hyperbolic space through the introduction of new classes of Morse functions, L_π (directed distance from a hyperplane) and L_h (directed distance from a horosphere). They proved the following theorem.

THEOREM A. *Let M^n, $(n \geqq 2)$, be a connected, complete Riemannian manifold isometrically immersed in H^m. Every Morse function of the form L_p or L_π has index* 0 *or n at all its critical points if and only if M^n is embedded as a sphere, horosphere or equidistant hypersurface in a totally geodesic $H^{n+1} \subset H^m$.*

In this paper, we shall study more general submanifolds in a hyperbolic space using Morse functions. For background material and notation, we refer the reader to [2].

AMS(MOS) 1970 *subject classification*: Primary 50C05, 58E05; Secondary 53A35, 53C40.

Key words and phrases: Morse function, index, eigenvalues of the second fundamental form.

Received by the editors on May 16, 1981, and in revised form on March 15, 1982.