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A CLASSICAL BANACH SPACE: l_∞/c_0

I.E. LEONARD[1] AND J. H. M. WHITFIELD[2]

ABSTRACT. This is an expository paper in which we study some of the structural and geometric properties of the Banach space l_∞/c_0 using its identification with $C(\beta\mathrm{N}\backslash\mathrm{N})$. In particular, it is noted that although l_∞/c_0 is not a dual space, its unit ball has an abundance of extreme points. Also, its smooth points are classified and its complemented subspaces are studied.

1. Introduction. The Banach space l_∞/c_0 certainly falls into the category of a "classical Banach space". Not only has it been around since the time of Banach's original monograph (1932) [2], but it is also classical in the sense of Lacey [17] or Lindenstrauss and Tzafriri [18] since it is congruent (isometrically isomorphic) to the space $C(\beta\mathrm{N}\backslash\mathrm{N})$. However, many of its interesting properties have not been as widely circulated as those of some of the other classical Banach spaces. In this paper, which is of an expository nature, it is our intention to begin to rectify this situation.

We will begin with some definitions. l_∞ is the linear space of bounded sequences of real numbers and c_0 is the subspace of sequences which converge to zero. Both of these spaces, when provided with the supremum norm, $\|x\| = \sup|x_n|$ where $x = \{x_n\}_{n\geqq 1}$, are Banach spaces. The quotient space l_∞/c_0 is the usual linear space of cosets $\hat{x} = x + c_0$, $x \in l_\infty$. When provided with the quotient norm $\|\hat{x}\| = \inf\{\|x - y\| : y \in c_0\}$, $\hat{x} = x + c_0$, it is a Banach space.

Although much is known about quotient spaces in general, this is not the appropriate method for studying l_∞/c_0. Indeed, much more is known about $C(T)$, the space of continuous real valued functions on the compact, Hausdorff space T, and we shall see shortly that l_∞/c_0 is congruent to $C(\beta\mathrm{N}\backslash\mathrm{N})$.

Recall that if T is a Tychonoff space, (i.e., completely regular and Hausdorff) then its Stone-Čech compactification βT can be described as

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