

REMARKS ON HYPONORMAL TRIGONOMETRIC TOEPLITZ OPERATORS

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ABSTRACT. Presented in this note are a characterization of hyponormal trigonometric Toeplitz operators of degree two and a characterization of operators (in the class of hyponormal trigonometric Toeplitz operators) whose self-commutators are of rank one.

For ϕ in L^∞ of the unit circle, the Toeplitz operator T_ϕ with symbol ϕ is the operator on H^2 of the unit disk defined by the equation $T_\phi(f) = P(\phi f)$ where P is the orthogonal projection onto H^2 . A trigonometric Toeplitz operator of degree n is a Toeplitz operator T_ϕ with $\phi(z) = \sum_{k=-n}^n a_k z^k$, where a_n or $a_{-n} \neq 0$. An operator T is called hyponormal if its self-commutator, $T^*T - TT^*$, is a positive operator.

It is obvious that, given $\phi(z) = a_1 z + a_{-1} \bar{z}$, T_ϕ is a hyponormal operator if and only if $|a_1| - |a_{-1}| \geq 0$. A less obvious fact is that, given $\phi(z) = \sum_{k=-n}^n a_k z^k$, the self-commutator of the hyponormal trigonometric Toeplitz operator T_ϕ of degree n is of rank zero (in other words, T_ϕ is normal if and only if $|a_n| = |a_{-n}| (>0)$); this fact was first observed by Ito and Wong [4, remark 4]. The purpose of this note is to present the following extensions which, we hope, may shed some light on the problem of characterizing hyponormal Toeplitz operators [1, problem 1], and on the problem of identifying operators (in the class of trigonometric Toeplitz operators) the ranks of whose self-commutators are given. These extensions also provide us with means to construct easily hyponormal operators that are not subnormal (for other types of examples, see: Halmos [3, problems 160 and 164]; Putnam [5, p. 60]; Fan and Stampfli [2]).

THEOREM 1. *Let $\phi(z) = \sum_{k=-2}^2 a_k z^k$, where a_2 or $a_{-2} \neq 0$. The trigonometric Toeplitz operator T_ϕ of degree two is hyponormal if and only if $|a_2|^2 - |a_{-2}|^2 \geq |a_2 \bar{a}_{-1} - \bar{a}_{-2} a_1|$.*

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