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PROPERTIES OF SHEPARD'S SURFACES

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ABSTRACT. Shepard's formula is an interpolation method for arbitrarily placed bivariate data. In this paper the continuity class and interpolation properties are proved. This is followed by generalizations which make the original method more useful. Graphical illustrations of the various methods conclude the paper.

1. Introduction. In this paper we consider the problem of interpolation to arbitrarily spaced data. Typically the problem arises when a surface model is required to interpolate scattered spatial measurements. This problem is encountered in such areas as geology, cartography, earth sciences and many others. One example would be to generate a surface model for a mineral deposit from data gathered at exploratory drillings.

The interpolation problem is given $\{(x_i, y_i, F_i)\}_{i=1}^n$ find a surface function G so that $G(x_i, y_i) = F_i$, $i = 1, 2, \ldots, n$. Different methods for solving this problem are the following.

(1) Triangulation of the domain followed by the appropriate triangular interpolant [**1**].

(2) Preprocess the data so that procedures requiring rectangularly gridded data are applicable [**7**].

(3) Shepard type methods [**1**, **3**, **5**, **8**].

The Shepard type method is the focal point of this paper. What we propose are methods that not only interpolate to positional information but allow the interpolation to specified derivatives at the scattered points (x_i, y_i). The methods we propose do not necessarily require higher order derivatives, but we make the option of supplying more general interpolation data available to a user. Figure 1, 2, 4, and 5 are four surfaces which interpolate the same (arbitrarily specified) positional information. These four surfaces are quite different because of the preprocessor and the derivatives used in the interpolation.

In the next section we discuss the mathematics that leads us to the

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