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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO PERTURBED LINEAR DIFFERENTIAL EQUATIONS

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

Introduction. The purpose of this paper is to investigate the asymptotic relationship between solutions of the n-th order linear homogeneous equation

$$L_n y = 0 \tag{1}$$

and those of the perturbed equation

$$L_n y + B(x, y, y', \ldots, y^{(n-1)}) = 0. \tag{2}$$

The results will involve certain smallness conditions on the function $B(x, y, y', \ldots, y^{(n-1)})$ which will be made more precise in later sections. In the first section we will consider the general case where $L_n y$ admits a Mammana factorization [**6**]. In the second section we shall consider the case where $L_n y$ is a constant coefficient operator. In the third section we shall consider the specific operator $L_n y = y^{(n)}$. This section also contains examples to show the results obtained here generalize those of Svec [**7**], [**8**], and Belohorec [**3**].

I. Perturbed linear equations. Mammana [**6**] has shown that, under certain conditions, an n-th order linear differential operator with leading coefficient one admits a factorization of the form

$$L_n[y] = \left(\prod_{j=1}^{n} (D - \eta_j(x)\right)[y] \tag{3}$$

where $\eta_j(x) = D[\ln\ W_j/W_{j-1}]$, $1 \leqq j \leqq n$, and W_j is the Wronskian of the solutions $\xi_1, \xi_2, \ldots, \xi_j (W_0 \equiv 1)$ of (1). The solutions $\xi_1, \xi_2, \ldots, \xi_n$ have the property that for every j, W_j is different from zero, which requires, in general, that the ξ_j be complex and hence the $\eta_i(x)$ will be complex. Levin [**5**] has observed the interval on which this holds may be half-line of the form $[a, \infty)$.

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