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GENERALIZED CONTINUOUS AND HYPERCONTINUOUS LATTICES

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A class of complete lattices which have recently received a considerable deal of attention is the class of continuous lattices introduced by D. Scott **[13]** (see also [3]). One of the interesting features of this class of lattices is the fact that these lattices admit a unique compact Hausdorff topology for which the meet operation is continuous (i.e., they admit the structure of a compact topological semilattice). This topology turns out to be an "intrinsic" topology, i.e., one that can be defined directly from the lattice structure. We refer to this topology as the CL-topology.

A major goal of this paper is to give a more detailed examination of this CL-topology. For any complete lattice this topology is always compact and T_1. We characterize those complete lattices for which it is Hausdorff; because these lattices have many characteristics reminiscent of continuous lattices, we call them generalized continuous lattices. They seem to be an interesting class of lattices in their own right; hence we develop some of their fundamental properties.

One of the oldest of the intrinsic topologies is Frink's interval topology. We address ourselves to the question of for what continuous lattices do the CL-topology and the interval topology coincide. This turns out to be precisely the class of lattices which we call "hypercontinuous". We turn our attention to these and point out some surprising connections between these lattices and generalized continuous lattices.

0. **Preliminaries.** In this preliminary section we collect some well-known notations, definitions and results needed later on.

DEFINITION 0.1. If L is any lattice and if $A \subseteqq L$ is a subset of L, then A is called on *upper set* provided that for $a, b \in L$, $a \leqq b$ and $a \in A$ implies $b \in A$. If A is any subset of L, then we denote by $\uparrow A$ the smallest upper set of L which contains A, i.e., $\uparrow A = \{x\colon \text{there is an } a \in A \text{ with } a \leqq x\}$. *Lower sets* and $\uparrow A$ are defined dually. An upper set (lower set), which is at the same time a sublattice of L is called a *filter* (*ideal*). A subset $I \subseteqq L$ is called *down-directed* (*up-directed*), if for each pair of elements a, $b \in I$

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