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A SINGULAR NONLINEAR BOUNDARY VALUE PROBLEM

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We consider the singular non-linear boundary value problem

$$\ddot{y} + \frac{\gamma}{t}\dot{y} - y + f(y^2)y = 0, \quad t \in (0, \infty) \tag{1.1}$$

$$\lim_{t\to 0+} y(t) > 0, \lim_{t\to\infty} y(t) = 0, \lim_{t\to 0+} \dot{y}(t) = 0, \tag{1.2}$$

where $1 \leqq \gamma \leqq 2$. It is shown that for certain functions f, positive in $(0, \infty)$ and continuous in $[0, \infty)$, the equation (1.1) has solutions $y_n(t)$, $n = 0, 1, 2, \ldots$, which satisfy (1.2) and vanish at n distinct points in $(0, \infty)$.

The problem is motivated by a model for stationary self-focusing of light beams given by Zakharov, Sobolev, and Synakh **[12]**; and others. After some simplification, their equation becomes

$$\ddot{y} + \frac{1}{t}\dot{y} - y + f(y^2)y = 0. \tag{1.3}$$

Of particular interest is the case $f(s) = s$, in which case (1.3) becomes

$$\ddot{y} + \frac{1}{t}\dot{y} - y + y^3 = 0. \tag{1.4}$$

Ryder **[11]** and Macki **[6]** have considered the equation

$$\ddot{x} - x + xF(x^2, t) = 0,$$

which under the substitutions

$$F(x^2, t) = f(x^2/t^2), \; y(t) = t^{-1}x(t)$$

becomes our equation (1.1) with $\gamma = 2$. The range $1 \leqq \gamma < 2$ is not included, and our condition (III) on the nonlinearity is different from theirs, so that neither result is contained in the other even for $\gamma = 2$. Nehari **[10]** has considered the equation

$$\ddot{y} + \frac{2}{t}\dot{y} - y + y^3 = 0, \tag{1.5}$$

which is also included in (1.1) for $\gamma = 2$. The thrust of this paper is to

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