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# DISFOCALITY AND NONOSCILLATORY SOLUTIONS OF $N$-TH-ORDER DIFFERENTIAL EQUATIONS

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In this paper we shall study various disfocality properties and their consequences on solutions of the differential equation

$$y^{(n)} + py = 0, \tag{E}$$

where $p$ is continuous and of constant sign on $[a, \infty)$. Equation (E) is said to be *disfocal* on an interval $I$ if, for every nontrivial solution $y$ of (E), at least one of the functions $y, y', \ldots, y^{(n-1)}$ does not vanish on $I$. If equation (E) is not disfocal on $I$, then there exists an integer $k(1 \leqq k \leqq n-1)$, a pair of points $b, c \in I$, $b < c$, and a nontrivial solution $y$ of (E) such that $k$ of the functions $y, y', \ldots, y^{(n-1)}$ vanish at $b$ and the remaining $n-k$ functions at $c$, i.e.,

$$\begin{aligned} y^{(j_i)}(b) &= 0, i = 0, 1, \ldots, k-1, \\ y^{(j_i)}(c) &= 0, i = k, \ldots, n-1, \end{aligned} \tag{1}$$

$$0 \leqq j_0 < j_1 < \cdots < j_{k-1} \leqq n-1, 0 \leqq j_k < j_{k+1} < \cdots < j_{n-1} \leqq n-1.$$

Here, $n-k$ is even or odd according as $p < 0$ or $p > 0$ **[10]**, which is the well-known parity condition that every nontrivial solution of the problem (E)-(1) must satisfy. Equation (E) is said to be $(j_0, j_1, \ldots, j_{k-1}) - (j_k, \ldots, j_{n-1})$ *disfocal* on an interval $I$ if for every pair of points $b$ and $c$ in $I$, $b < c$, the only solution satisfying the conditions in (1) is the trivial solution; furthermore, if $j_i = i$, $i = 0, 1, \ldots, n-1$, it is said to be $k-(n-k)$ *disfocal*, and this special case has been investigated by Nehari **[10, 11, 12]** and Elias **[2, 3, 4]**. We shall say that equation (E) is *eventually* $(j_0, j_1, \ldots, j_{k-1}) - (j_k, \ldots, j_{n-1})$ *disfocal* on $[a, \infty)$ if there exists a point $b \geqq a$ such that (E) is $(j_0, j_1, \ldots, j_{k-1}) - (j_k, \ldots, j_{n-1})$ disfocal on $[b, \infty)$. The concept of eventual $(j_0, j_1, \ldots, j_{k-1}) - (j_k, \ldots, j_{n-1})$ disfocality is related to the existence of nonoscillatory solutions satisfying a set of sign conditions as shown in Lemma 2. On the other hand, Lemma 1 states that only certain sets of sign conditions are admissible for nonoscillatory solutions of (E). Since the admissible sign conditions strongly depend on the parity of $n$ and the sign of $p$, it is convenient to consider the following four cases:

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