ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 9, Number 3, Summer 1979

RANDOM SETS AND CONFIDENCE PROCEDURES

WILLIAM A. BARNETT*

ABSTRACT. The set of confidence procedures is identified as a fundamental subset of the set of random sets. This is accomplished by defining a simple σ-field on the codomain of a set-valued mapping such that the mapping is a confidence procedure if it is measurable relative to that σ-field. The confidence procedure becomes Borel measurable when the σ-field is generated by a topology that we also define. The resulting topology and σ-fields are shown to be natural choices on $\mathscr{P}(R^n) - \{\phi\}$. The motivation for the approach is the same as that motivating the use of statistics in point estimation. The theory of confidence procedures in set estimation is shown to be a simple extension of the theory of statistics in point estimation.

1. **Introduction.** Confidence sets are defined to be realizations of random sets called confidence procedures. Hence the random sets of set estimation correspond naturally to the random variables of point estimation. While the simple definition of confidence sets used in elementary applied statistics suffices to describe those set-valued mappings relevant to elementary statistics, a definition of confidence sets appropriate to measure-theoretic formulations should characterize confidence procedures in general. The purpose of this paper is to provide such a characterization and to illustrate that the resulting confidence procedures are particularly simple and fundamental measurable set-valued mappings. The method adopted presents the theories of confidence procedures and random sets as extensions to the theories of statistics and random variables.

A step in the direction followed by this paper was taken by Wallace [13], who characterized a subset of the set of confidence procedures in terms of the properties of the mappings' graphs. Joshi's ([3], [4], [5], [6], [7]) recent work investigating the admissibility of common confidence procedures has demonstrated the usefulness of Wallace's definition in systematic studies of confidence procedures. Similarly Stein [12] has

Received by the editors on September 4, 1975, and in revised form on March 12,1977.

*This research was initiated in the Department of Statistics at Carnegie-Mellon University. Completion of the research was supported by the Board of Governors of the Federal Reserve System in Washington, D.C. The author would like to thank Professor Paul Shaman and one of the Editors for their helpful comments.

AMS(MOS) 1970 *Subject classifications*: 28A05, 62F25, 54A05.

Key words and phrases: neighborhood system topology, confidence procedures, random sets.