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ON A GENETICS MODEL OF MORAN EVOLVING IN RANDOM ENVIRONMENTS

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SUMMARY. In a previous investigation [6], a model of a discrete-time stochastic process (Z_n) evolving in a random environment controlled by an irreducible Markov chain (Y_n) was formulated wherein the bivariate process (Y_n, Z_n) is Markovian and the marginal process (Z_n) is a birth and death chain when conditioned on a fixed sequence of environmental states of (Y_n). Conditions for the extinction and instability of (Z_n) were stated and proved. In a succeeding investigation [7] methods were obtained to calculate extinction probabilities when the probability of extinction is less than one and (Y_n) has finite state space. In this paper, these methods are applied to a genetics model of [4] to study gene fluctuations due to mutation influences subject to varying environmental conditions.

1. **Introduction.** Consider a bivariate stochastic process (Y_n, Z_n), $n = 0, 1, 2, \cdots$ with state space $S_0 = \{1, \cdots, m\} \times \mathbf{Z}_0$ where $\mathbf{Z}_0$ denotes the non-negative integers. In a previous investigation (Torrez [6]), we formulated a mathematical model to represent a discrete-time birth and death process evolving in a random environment in such a way that (i) the marginal process (Y_n) of (Y_n, Z_n) (called the environmental process or simply environment) is an irreducible Markov chain with state space $\{1, \cdots, m\}$ and transition kernel K; (ii) given a realization of (Y_n), the conditional distribution of (Z_n) is Markovian (but not time-homogeneous, in general). Indeed, when a sequence of environmental states (y_n) is given, the marginal process (Z_n) of (Y_n, Z_n) behaves like a birth and death process with transition probabilities

$$Pr[Z_{n+1} = z' \mid (Y_n, Z_n) = (y_n, z_n)] =$$

$$(1.1)\qquad \begin{cases} p_{z_n}^{(y_n)} & \text{if } z' = z_n + 1, \\ q_{z_n}^{(y_n)} & \text{if } z' = z_n - 1 \\ r_{z_n}^{(y_n)} & \text{if } z' = z_n \\ 0 & \text{otherwise} \end{cases}$$

where

$$0 \leqq p_{z_n}^{(y_n)}, q_{z_n}^{(y_n)}, r_{z_n}^{(y_n)} \leqq 1,$$

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