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ON p'-AUTOMORPHISMS OF ABELIAN p-GROUPS

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All groups in this article are finite. Our notation is standard. In particular, let p denote an arbitrary prime integer, let P denote an arbitrary abelian p-group, let F denote the field $Z/(p)$ and let A be a p'-subgroup of $\mathrm{Aut}(P)$.

This basic situation is discussed, for example, in [1, § 5.2] and in [2, I, Aufgaben 68–69]. These references show that if P is A-indecomposable, then P is homocyclic. However, it is possible to extend this result to:

THEOREM. *P is A-indecomposable if and only if A acts irreducibly on $\Omega_1(P)$.*

Before describing a proof of this result, we present two applications.

First suppose that P is A-indecomposable. Thus P is homocyclic. Let $\exp(P) = p^n$. Then, for each integer i with $1 \leqq i < n$, the endomorphism $\bar{i}$ of P defined by $\bar{i} : x \to x^{p^i}$ lies in the center of the ring $\mathrm{End}(P)$ and $\bar{i}$ induces an A-isomorphism $\tilde{i}$ of $P/\Phi(P)$ onto $\Omega_{n-i}(P)/\Omega_{n-i-1}(P)$. Hence each of the elementary abelian p-groups $\Omega_j(P)/\Omega_{j-1}(P)$ for $1 \leqq j \leqq n$ is A-isomorphic to $\Omega_1(P)$. Thus [1, 3.2.2, 5.1.4 and 5.3.2] yield:

COROLLARY 1. *If P is A-indecomposable with $\exp(P) = p^n$, then:*

(a) *$\{\Omega_i(P) \mid 0 \leqq i \leqq n\}$ is the set of A-invariant subgroups of P;*

(b) *every A-invariant subquotient of P is A-indecomposable;*

(c) *A acts faithfully and irreducibly on $\Omega_1(P)$ and $Z(A)$ is cyclic; and*

(d) *all A-composition factors of P are A-isomorphic to $\Omega_1(P)$.*

Next let P be arbitrary and let $\{V_i \mid 1 \leqq i \leqq s\}$ be a set of representatives for the distinct isomorphism types of irreducible representations of $F[A]$ where A acts trivially on V_1 and let

$$(*) \qquad P = P_1 \times P_2 \times \cdots \times P_r$$

be a direct decomposition of P into A-indecomposable subgroups. For $1 \leqq i \leqq s$, let Q_i be the (direct) sum of all P_j such that $\Omega_1(P_j)$ is $F[A]$-isomorphic to V_i. Clearly $Q_1 \leqq C_P(A)$, $[Q_j, A] = Q_j$ if $j > 1$, $[P, A] = \prod_{j=2}^{s} Q_j$ and $C_P(A) \cap \prod_{j=2}^{s} Q_j = 1$. Hence:

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