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STABLE HOMOTOPY AND ORDINARY DIFFERENTIAL EQUATIONS WITH NONLINEAR BOUNDARY CONDITIONS

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1. **Introduction.** This paper is a continuation of [1] where coincidence degree arguments have been used to give fairly general existence theorems for nonlinear boundary value problems relative to ordinary differential equations. In contrast with [1] where the case of nonlinear perturbations of Fredholm mappings of index zero has been treated, we consider here the case where this index is positive.

Following Nirenberg [5] we use stable homotopy arguments to get a continuation theorem which was announced in [4] and is given here with complete proof for reader's convenience (Section 2). This continuation result leads in Section 3 to a fairly general existence theorem for boundary value problems. An interesting specialization and an example are given in Section 4.

2. **A continuation theorem for some nonlinear perturbations of Fredholm mappings with non-negative index.** Let X and Z be real normed spaces and $L : \mathrm{dom}\, L \subset X \to Z$ a linear mapping such that $\mathrm{Im}\, L$ is closed and

$$q = \mathrm{codim}\,\mathrm{Im}\, L \leqq \dim \ker L = p.$$

We shall call L a *Fredholm mapping of index* $p - q$. Let $R > 0$ and $N : \bar{B}(R) \subset X \to Z$ *be L-compact* on the closed ball $\bar{B}(R)$ of center 0 and radius R. That means [4] that if $P : X \to X$ and $Q : Z \to Z$ denote continuous projectors such that the sequence

$$X \xrightarrow{P} \mathrm{dom}\, L \xrightarrow{L} Z \xrightarrow{Q} Z$$

is exact, and if

$$K_{P,Q} = (L \mid \mathrm{dom}\, L \cap \ker P)^{-1}(I - Q),$$

then QN is continuous on $\bar{B}(R)$, $QN(\bar{B}(R))$ is bounded and $K_{P,Q}N : \bar{B}(R) \to X$ is compact. It is known [4] that those conditions are independent of the choice of P and Q. If now

$$\Gamma : R^p \to \ker L, \Gamma' : \mathrm{Im}\, Q \to R^q$$

are isomorphisms, we shall define the mapping ν by

$$\nu(u) = \frac{\Gamma' QN\Gamma(Ru)}{|\Gamma' QN\Gamma(Ru)|} \tag{2.1}$$