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COHOMOLOGY OF QUASI-PROJECTIVE STIEFEL MANIFOLDS

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1. **Introduction.** Let $V_{n,k}$ denote the Stiefel manifold of orthonormal k-frames in Euclidean n-space. The special orthogonal group $SO(2)$ acts freely on $V_{2n,k}$ via the diagonal embedding of S^1 in $U(n)$ and the standard embedding of $U(n)$ into $SO(2n)$ corresponding to realification $r: BU(n) \to BSO(2n)$. The quasi-projective Stiefel manifold $PV_{2n,k}$ is the quotient space of $V_{2n,k}$ under this action of $SO(2)$. The spaces $PV_{2n,k}$ are classifying spaces for sectioning multiples of a complex line bundle. If X is a finite complex and ξ a complex line bundle over X, then $n\xi$ has k linearly independent real sections if and only if there is a map $f: X \to PV_{2n,k}$ such that $f^*\eta_0 = \xi$ where η_0 is the complex line bundle over $PV_{2n,k}$ associated to the S^1-fibering $V_{2n,k} \to PV_{2n,k}$. In this paper we determine the cohomology algebras of the spaces $PV_{2n,k}$.

2. **Preliminaries.** We first establish some notation. Let $RE(x_i \mid i \in I)$ denote the exterior algebra over a ring R with generators x_i of degree i. Let $V(x_1, \cdots, x_m)$ denote the commutative associative algebra over Z_2 on generators $x_1, \cdots, x_m$ such that the monomials $x_1^{\epsilon_1} \cdots x_m^{\epsilon_m}$ with $\epsilon_i = 0$ or 1 form an additive basis. Let $\{{}_pE_r(X)\}$ denote the mod p Bockstein spectral sequence for X with ${}_pE_1(X) = H^*(X; Z_p)$. $C_{r,i}$ denotes the binomial coefficient $\binom{r}{i}$. Let ρ_p denote the universal coefficient map $H^*(\ ; Z) \to H^*(\ ; Z_p)$ for any prime p and let ρ_0 denote the map $H^*(\ ; Z) \to H^*(\ ; Q)$. Denote the image of an integral class x under the projection $H^*(X; Z) \to H^*(X; Z)/\text{Tors}$ by $\bar{x}$. Finally let $J_{n,k}$ represent the set of all integers j such that $[(2n-k)/2] < j < n$ where $0 < k < 2n$. We write $H^*(CP^\infty) = Z[\beta]$.

Recall from [5] the cellular structure of the Stiefel manifold $V_{2n,k}$ obtained from an embedding of real projective space RP^{2n-1} into $O(2n)$ composed with the projection map $O(2n) \to V_{2n,k}$. The image of RP^{2j} determines a class P^{2j} in $H^{2j}(V_{2n,k}; Z)$ of order 2 for every $j \in J_{n,k}$. *Set* $x_{2j} = \rho_2(P^{2j})$. RP^{2n-k} determines a free integral class y_{2n-k} for k even. Let $x_{2n-k} = \rho_2(y_{2n-k})$ for k even and let x_{2n-k} be the unique class such that $\mathrm{Sq}^1 x_{2n-k} = x_{2n-k+1}$ for k odd. By [2]

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