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FUNCTIONS ANALOGOUS TO COMPLETELY CONVEX FUNCTIONS[1]

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ABSTRACT. We consider the problem of expanding a function $f \in C^\infty[0, 1]$ in a $L_{3,0,0}$ series, which is an analogue of Lidstone series. To this end we are led to consider the class $W_{3,0,0}$ of functions and the class of minimal $W_{3,0,0}$ functions. Following Widder's method for completely convex functions, we show that a function f has an absolutely convergent $L_{3,0,0}$ series expansion if and only if $f = g - h$, where g, h belong to the class of minimal $W_{3,0,0}$ functions. The existence of five more similar results is pointed out.

1. **Introduction.** Recently Leeming and Sharma [3] have given a generalization of completely convex functions by considering an analogue of Lidstone series. In 1942, Widder [9] showed the close connection of completely convex functions with Lidstone series, similar to the one that exists between the completely monotonic functions of Bernstein and the Taylor series. For details we refer the reader to [10]. In 1942 many deep studies were made in connection with "the influence of the sign of the derivatives of a function on its analytic character", which seem to have deep connections with Widder's class of completely convex functions. In particular Boas and Pólya [1] showed, roughly speaking, that if $\{n_k\} \uparrow$ and $\{q_k\}$ are two sequences of nonnegative integers and if

$$(1.1) \qquad f^{(n_k)}(x) f^{(n_k + 2q_k)}(x) \leqq 0 \quad \text{on a given interval } I,$$

then f must coincide on I with an entire function of order 1 and finite type. Although this result is of a very general nature, and although it extends one of the results of Widder considerably, the other interesting results of Widder on completely convex functions went almost unnoticed.

Our object here is to follow the point of view of Widder [9] and to consider a two point interpolation problem analogous to Lidstone interpolation. We shall use the notation of an incidence matrix for an

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