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MULTIPLE SUBDIVISIONS OF E^n

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1. **Preliminaries.** The notion of an asymptotic subdivision of Euclidean n-space, E^n, was introduced by Groemer [1]. His work was suggested by a result of Schmidt [4]. An asymptotic subdivision may be thought of as a configuration of sets in which the ratio of that part of E^n which is not covered by exactly one set of the configuration to the whole of E^n is 0. In particular, packings and coverings of density 1 are asymptotic subdivisions. (For further discussion of these ideas, the interested reader is referred to Groemer [2] and Rogers [3].) A stronger requirement on this configuration of sets is that almost every point of E^n, lies in exactly one set from the configuration. Such a configuration is called a subdivision of E^n. Under very general hypotheses, Groemer showed that from a given asymptotic subdivision of E^n one can obtain a related subdivision of E^n. In this paper, these ideas will be generalized to the case in which points of E^n lie in exactly k sets (where k is an arbitrary integer greater than 1). We shall refer to such as "k-subdivisions".

We shall consider sequences of sets rather than families of sets in discussing k-subdivisions. This is necessary because some sets may occur repeatedly and should be considered as different. The order in which the sets occur in the sequence is immaterial. (Generalization to uncountable "sequences" of sets may be made, but we shall not do that here.)

If only a finite number of these sets lie in any bounded region of E^n, we say the sets do not accumulate; otherwise, we say the sets *accumulate*.

If $\{X_i\}$, $X_i \subset E^n$, is a (countably) infinite sequence, we shall denote a subsequence of $\{X_i\}$ by $\{X_{ij}\}$ rather than by $\{X_{i_j}\}$. This notation will be employed for the remainder of this paper.

If X is a Lebesgue measurable set in E^n, then $m(X)$ denotes the *measure* of X and $d(X)$ denotes the *diameter* of X.

A sequence S of bounded sets in E^n each of which has positive measure is said to be *compact in the topology of symmetric differences* if to every infinite sequence $\{X_j\}$ in S there exists a subsequence $\{X_{ji}\}$

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