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ON DECOMPOSITIONS OF $E(G)$[1]

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1. **Introduction.** The theory of near rings has been studied in some detail by several authors. In a paper that briefly summarized the elementary theory of near rings Berman and Silverman [1] generalized the Peirce Decomposition Theorem to obtain a decomposition theorem for near rings. Fröhlich [2], [3] studied the class of distributively generated near rings, and Malone [4] has emphasized the class of endomorphism near rings.

For an arbitrary group G the set of endomorphisms of G, denoted by End(G), form a distributive generating set (d.g. set) for the endomorphism near ring $E(G)$. The convention of writing functions on the right (i.e., $f: G \to G$ sends g to $(g)f$) makes $E(G)$ a left near ring. Therefore, all of the results in this paper are stated for left near rings.

The decomposition of Berman and Silverman provides a starting point for the investigation of two basic problems related to endomorphism near rings. First, by examining the decomposition theorem and using a construction technique of Malone and Lyons [6] one is able to construct classes of groups for which the endomorphism near ring decomposes in a predictable manner. Secondly, one is able to supply a sufficient condition on the relationship between groups G and H so that $E(H)$ embeds in $E(G)$. This provides an embedding result for endomorphism near rings that parallels the results of Malone and Heatherly [5] for the embedding of transformation near rings.

2. **The decomposition.** The statement of the Berman and Silverman decomposition theorem is

THEOREM 2.1 [1, p. 27]. *Let e be an idempotent in the near ring R. For each $r \in R$, $r = er + (-er + r) = (r - er) + er$. Thus $R = A_e + M_e = M_e + A_e$ where $A_e = \{r - er : r \in R\} = \{t \in R : et = 0\}$, $M_e = \{er : r \in R\}$, and $A_e \cap M_e = \{0\}$.*

When no confusion can arise the summands A_e and M_e will be designated by A and M respectively.

Theorem 2.1 says that the group structure of any near ring with

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[1]Portions of this paper appear in the author's Ph.D. dissertation written under the direction of Professor J. J. Malone, Jr. at Texas A & M University.