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BIFURCATION IN SINGULAR SELFADJOINT BOUNDARY VALUE PROBLEMS[1]

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1. **Introduction.** In recent years considerable progress has been made in the study of the bifurcation phenomenon associated with non-linear perturbations of Sturm-Liouville problems. For example,

$$\begin{aligned} &Ly = \lambda[y + f(t, y)], \\ (1.1) \qquad &\alpha y(0) + \beta y'(0) = 0, \\ &\gamma y(1) + \delta y'(1) = 0, \end{aligned}$$

where $Ly = -(py')' + qy$ and f is small in the appropriate sense as $y \to 0$. A typical result is: if λ_k is an eigenvalue of the linearized problem (i.e. $f \equiv 0$), there exist solutions to the nonlinear problem (1.1) for small $y \not\equiv 0$ and $(\lambda - \lambda_k)$. The result is considered as a branching or bifurcation from the point $(\lambda_k, 0)$ relative to the sub-space $\{(\lambda, y) : y = 0\}$. For example, see [1] and [2].

More recently it has been shown that, although the bifurcation phenomenon is usually considered to be a local result, it often is global in the sense that the solution pair to (1.1), (λ, y), can be extended indefinitely; i.e. $\|(\lambda, y)\| \to \infty$. Some results along this line have been established using topological techniques by Crandall and Rabinowitz [3] and Turner [4].

Our purpose here is to establish a *local* bifurcation property for a generalization of (1.1) which will include singular problems on intervals of the form $[0, \omega)$ where $\omega \leqq \infty$. The right boundary condition of (1.1) is replaced by: $y \in D$, D a Banach space. This condition is motivated by singular conditions like $D = L^\infty(0, \omega)$ or $D = L^2(0, \omega)$. We consider

$$(1.2a) \qquad L_0 y = -(ay')' + b_0 y = \lambda[y + f(t, y)],$$

$$(1.2b) \qquad m_0 y(0) - y'(0) = 0,$$

$$(1.2c) \qquad y \in D,$$

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