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A HOMOMORPHISM OF A PSEUDO PLANE ONTO A PROJECTIVE PLANE

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1. The purpose of this paper is to give an example of a homomorphism of a proper pseudo plane onto a projective plane. The pseudo plane used is coordinatized by Zemmer's nonplanar nearfield [5], and the image plane is a field plane. With the exception of the concept of place of a homomorphism the notation and terminology will follow that found in [4]. In §2 we give a characterization of place found in [1], and in §3 we give the example referred to above.

2. We let π and π' be pseudo planes and $\alpha : \pi \to \pi'$ be a homomorphism. We may choose a coordinatizing quadrangle for π such that its image is a coordinatizing quadrangle for π'. Call the quadrangle for π, (∞), (0), $(0, 0)$, $(1, 1)$. Let T and T' be the pseudo ternaries associated with these quadrangles for π and π' respectively. Pseudo ternaries are discussed in [3]. Then there is a mapping $\bar{\alpha} : T \to T' \cup \{\infty\}$ defined by

$$\bar{\alpha}b = \begin{cases} b' & \text{if } \alpha(0, b) = (0', b'), \\ \infty & \text{if } \alpha(0, b) = \alpha(\infty). \end{cases}$$

$\bar{\alpha}$ is called a place of α. Generally no confusion results from denoting $\bar{\alpha}$ by α.

If we assume that $(T, +, \cdot)$ is a nearfield, then the proof of Theorem 4.3 found in [1] suffices to show that α is a place of a homomorphism if and only if the following hold:

S1. $\alpha 0 = 0$, and $\alpha 1 = 1$.

S2. αa and $\alpha b \neq \infty$ implies $\alpha(a + b) = \alpha a + \alpha b$, and $\alpha(ab) = \alpha a \alpha b$.

S3. $\alpha a \neq \infty$ and $\alpha b = \infty$ implies $\alpha(a + b) = \alpha(b + a) = \infty$.

S4. $\alpha a \neq 0$ and $\alpha b = \infty$ implies $\alpha(ab) = \alpha(ba) = \infty$.

S5. $\alpha(-ax + a^*x) \neq \infty$ and $\alpha x = \infty$ implies $\alpha a = \alpha a^*$.

S6. $\alpha(ax - ax^*) \neq \infty$ and $\alpha a = \infty$ implies $\alpha x = \alpha x^*$.

S7. $a^*x + ax^* = ax$ and $\alpha a = \alpha x = \alpha(a^*x) = \alpha(ax^*) = \infty$ implies $\alpha a^* = \infty$ or $\alpha x^* = \infty$.

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