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OSCILLATION PROPERTIES OF THIRD ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. Oscillation properties of elements of possible bases for the solution space of a third order linear differential equation are considered.

1. **Introduction.** We will consider the differential equation

$$y''' + p(x)y' + q(x)y = 0 \tag{1}$$

and its adjoint

$$y''' + p(x)y' + (p'(x) - q(x))y = 0, \tag{2}$$

where we will assume that the coefficients are continuous on $[0, +\infty)$. In particular, we will consider equations which are of Class I or Class II as defined by Hanan [1].

We will consider a solution of (1) oscillatory if it changes sign for arbitrarily large x.

It has been shown by Utz [3], that the solution space of equation (1) can have at the same time a basis consisting of i oscillatory solutions and $3 - i$ nonoscillatory solutions, for $i = 0, 1, 2, 3$.

We will describe the types of bases possible for the solution spaces of equations (1) of Class I and Class II, with respect to the number of oscillatory solutions possible in a given basis. In doing so, we will generalize a theorem of Utz [3].

2. An equation (1) is said to be Class I if any solution for which $y(a) = y'(a) = 0$, $y''(a) > 0$ is positive on $[0, a)$. It is said to be Class II if any solution for which $y(a) = y'(a) = 0$, $y''(a) > 0$ is positive on $(a, +\infty)$. It was shown by Hanan [1] that (1) is Class I if and only if (2) is Class II.

In [1], Hanan considers a solution $y(x)$ of (1) to be oscillatory if it has an infinity of zeros in $[0, +\infty)$, but it follows from the definitions that if (1) is Class I or Class II, then this definition of oscillation implies $y(x)$ must change signs for arbitrarily large x.

We will use a method similar to that used by Lazer [2, p. 437] to prove the following lemma.

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