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MORE ON COMPLEMENTS OF MINIMAL SPANNING SURFACES

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ABSTRACT. W. R. Alford in volume 91 of the Annals of Mathematics has shown the existence of a knot which has two minimal spanning surfaces whose complements in S^3 are not homeomorphic. The trefoil knot is a companion to the knot. This paper shows that any nontrivial knot k is a companion to a knot K which has at least two minimal spanning surfaces.

Introduction. In [1], W. R. Alford exhibited a knot k and two minimal spanning surfaces S_1 and S_2 for k such that $S^3 - S_i$ are not homeomorphic. The knot was formed by sending the torus T containing the knot l in Fig. 1 faithfully to a regular neighborhood of the trefoil knot.

In a later paper [2], Alford and C. B. Schaufele constructed knots with 2^m really distinct minimal spanning surfaces; the surfaces do not have homeomorphic complements. The examples were constructed by sending the torus T containing the knot l in Fig. 1 faithfully to a regular neighborhood of the sum of m "nice" knots. The selection of the knots was strongly influenced by their algebraic properties.

The purpose of this paper is to show that any nontrivial knot is a companion to a knot K which has at least two minimal surfaces.

The knot K is the image of the knot l in T in Fig. 1 under a faithful homeomorphism of the solid torus T to a regular neighborhood V of the knot l.

The Alexander polynomial of K is $(2 - t) \cdot (2t - 1)$ [4] for any nontrivial k used. Thus K had genus at least one. The spanning surfaces for K have genus one, so K has genus one.

The surfaces. The surfaces for K are constructed as in [2]. The knot l is spanned by a singular disk in T as shown in Fig. 2. Only one side is shown; the singularities are in heavy lines.

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