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OBSTRUCTIONS TO EMBEDDING AND ISOTOPY IN THE METASTABLE RANGE

LAWRENCE L. LARMORE

1. Introduction.

1.1. *Preliminary definitions and summary.* Throughout this paper, "manifold" means differentiable manifold (closed or open) without boundary, with a countable base. "Differentiable" means infinitely differentiable, and "embedding" means differentiable embedding.

Suppose V and M are manifolds of dimension k and n, respectively, V compact, and $f : V \to M$ is a differentiable map. An *embedding homotopy* of f (abbreviated e-homotopy) shall be defined to be a homotopy of differentiable maps, $f_t : V \to M$, for $0 \leqq t \leqq 1$, such that $f_0 = f$ and f_1 is an embedding. We say that e-homotopies $\{f_{0,t}\}$ and $\{f_{1,t}\}$ are *isotopic* if there exists a 2-parameter homotopy of differentiable maps $f_{\tau,t} : V \to M$, for $0 \leqq \tau,\ t \leqq 1$, such that $f_{\tau,0} = f$ and $f_{\tau,1}$ is an embedding for all τ. Let $[f_t]$ denote the isotopy class of $\{f_t\}$, and let $[V \subset M]_f$ denote the set of all isotopy classes of e-homotopies of f.

It is not difficult to show that if f is an embedding, $[V \subset M]_f$ naturally has the structure of an Abelian group with identity $[f]$ (where $\{f\}$ is the constant homotopy), provided $2n > 3(k + 1)$. However, this construction is not within the scope of the present paper; we refer the reader to J. C. Becker [1] for the case when M is a Euclidean space. $[V \subset R^n]_f$ becomes $E(V, n)$, the so-called embedding group.

We consider three problems in this paper. The first is existence of an e-homotopy of f, i.e., whether $[V \subset M]_f$ is nonempty; the second is enumeration of $[V \subset M]_f$; more precisely, whether two given e-homotopies are isotopic. The third question deals with the function $\Delta : [V \subset M]_f \to [V \subset M]$, where $[V \subset M]$ is the set of isotopy classes of embeddings of V into M, and where, for any e-homotopy $\{f_t\}$ of f, $\Delta[f_t] = [f_1]$, the isotopy class containing f_1. As we see in §3.5, there is an action of $\pi_1(M^V, f)$ on $[V \subset M]_f$ whose orbits correspond to the image of Δ, where M^V is the space of differentiable functions $V \to M$ with the compact-open topology. In §3.8, we discuss

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