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SOME ASPECTS OF NONLINEAR EIGENVALUE PROBLEMS[1]

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Introduction. Let E be a real Banach space and $G: R \times E \to E$ where G is continuous. Consider the equation

$$u = G(\lambda, u) \tag{0.1}$$

where $\lambda \in R$ and $u \in E$. A *solution* of (0.1) is a pair $(\lambda, u) \in R \times E$. Equations of the form (0.1) are generally called *nonlinear eigenvalue problems*. As has been amply demonstrated at this symposium, they occur in many parts of mathematical physics. Our main interest here is in studying the structure of the set of solutions of (0.1). We restrict ourselves to real λ and to real Banach spaces since this is the situation usually encountered in applications.

Our survey will focus on two major approaches to the study of nonlinear eigenvalue problems, namely the theory of topological degree of Leray and Schauder and the theory of critical points of Ljusternik and Schnirelmann. The applications of degree theory will be mainly to bifurcation problems although some results will be given for the case where bifurcation need not occur. A very general treatment of the Ljusternik-Schnirelmann theory has been presented in his lectures by Professor Browder. Here a simpler version will be given for a very special case — namely the manifolds dealt with will be spheres in a Hilbert space — our goal being to bring out the underlying ideas in essentially as simple a context as possible.

In §1 applications of degree theory to bifurcation theory will be developed. In particular a generalization of a theorem of Krasnoselski is obtained showing that in a certain context bifurcation is a global phenomenon. A useful constructive local theorem for bifurcation from simple eigenvalues is also given. Several applications to ordinary and partial differential equations of the results of §1 are carried out in §2.

The notion of genus is introduced in §3 and its properties studied.

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