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ON THE GENERALIZED DISCREPANCY PRINCIPLE FOR TIKHONOV REGULARIZATION IN HILBERT SCALES

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ABSTRACT. For solving linear ill-posed problems regularization methods are required when the right hand side and the operator are with some noise. In the present paper regularized solutions are obtained by Tikhonov regularization in Hilbert scales and the regularization parameter is chosen by the generalized discrepancy principle. Under certain smoothness assumptions we provide order optimal error bounds that characterize the accuracy of the regularized solution. It appears that for getting small error bounds a proper scaling of the penalizing operator B is required. For the computation of the regularization parameter fast algorithms of Newton type are constructed which are based on special transformations. These algorithms are globally and monotonically convergent. The results extend earlier results where the problem operator is exactly given. Some of our theoretical results are illustrated by numerical experiments.

1. Introduction. In this paper we are interested in solving ill-posed problems

$$A_0 x = y_0, \tag{1.1}$$

where $A_0 \in \mathcal{L}(X, Y)$ is a linear, injective and bounded operator with non-closed range $\mathcal{R}(A_0)$ and X, Y are Hilbert spaces with corresponding inner products $(\cdot, \cdot)$ and norms $\|\cdot\|$. Throughout we assume that

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