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A NUMERICAL METHOD FOR A NONLOCAL ELLIPTIC BOUNDARY VALUE PROBLEM

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ABSTRACT. In 2005 Corrêa and Filho established existence and uniqueness results for the nonlinear PDE: $-\Delta u = \frac{g(x,u)^{\alpha}}{\left(\int_{\Omega} f(x,u)\right)^{\beta}}$, which arises in physical models of thermodynamical equilibrium via Coulomb potential, among others [3]. In this work we discuss a numerical method for a special case of this equation: $-\alpha\left(\int_0^1 u(t)dt\right)u'' = f(x), \quad 0 < x < 1, \quad u(0) = a, \quad u(1) = b$. We first consider the existence and uniqueness of the analytic problem using a fixed point argument and the contraction mapping theorem. Next, we evaluate the solution of the numerical problem via a finite difference scheme. From there, the existence and convergence of the approximate solution will be addressed as well as a uniqueness argument, which requires some additional restrictions. Finally, we conclude the work with some numerical examples where an interval-halving technique was implemented.

1. Introduction.

At the annual meeting of the American Mathematical Society in Baltimore in January 2003, the first named author above gave a talk at a special session organized by Zuhair Nashed. Part of the talk included an example of a boundary value problem which involved a coefficient that depended upon the integral of the solution over the domain within the differential equation. Namely,

$$u'' = \alpha\left(\int_0^{\infty} u(t)dt\right)u, \tag{1.1}$$
$$0 < x < \infty, \quad u(0) = 1, \quad \lim_{x\to\infty} u(x) = 0,$$

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