JOURNAL OF COMMUTATIVE ALGEBRA
Volume 4, Number 4, Winter 2012

ON THE CHERN NUMBER OF I-ADMISSIBLE FILTRATIONS OF IDEALS

MOUSUMI MANDAL AND J.K. VERMA

ABSTRACT. Let I be an $\mathfrak{m}$-primary ideal of a Noetherian local ring $(R, \mathfrak{m})$ of positive dimension. The coefficient $e_1(\mathcal{I})$ of the Hilbert polynomial of an I-admissible filtration $\mathcal{I}$ is called the Chern number of $\mathcal{I}$. A formula for the Chern number has been derived involving the Euler characteristic of subcomplexes of a Koszul complex. Specific formulas for the Chern number have been given in local rings of dimension at most two. These have been used to provide new and unified proofs of several results about $e_1(\mathcal{I})$.

Introduction. Let $(R, \mathfrak{m})$ be a Noetherian local ring of dimension d, and let I be an $\mathfrak{m}$-primary ideal. A sequence of ideals $\mathcal{I} = \{I_n\}_{n\in\mathbf{Z}}$ is called an I-*admissible filtration* if a $k \in \mathbf{N}$ exists such that, for all $n, m \in \mathbf{Z}$,

$$I_{n+1} \subseteq I_n, \quad I_m I_n \subseteq I_{n+m} \quad \text{and} \quad I^n \subseteq I_n \subseteq I^{n-k}.$$

The Rees algebra $\mathcal{R}(\mathcal{I})$ and the associated graded ring $G(\mathcal{I})$ of the filtration $\mathcal{I}$ are defined as

$$\mathcal{R}(\mathcal{I}) = \bigoplus_{n\in\mathbf{Z}} I_n t^n, \qquad G(\mathcal{I}) = \bigoplus_{n\geq 0} I_n/I_{n+1}.$$

For the I-adic filtration $\mathcal{I} = \{I^n\}$, we put $\mathcal{R}(\mathcal{I}) = \mathcal{R}(I)$ and $G(\mathcal{I}) = G(I)$. Note that $\mathcal{I}$ is an I-admissible filtration if and only if $\mathcal{R}(\mathcal{I})$ is a finitely generated $\mathcal{R}(I)$-module. Rees [**10**] proved that the integral closure filtration $\{\overline{I^n}\}$ is an I-admissible filtration if and only if R is analytically unramified. Marley [**9**] showed that if $\mathcal{I}$ is an I-admissible filtration, then the *Hilbert function* $H_{\mathcal{I}}(n) = \lambda(R/I_n)$, where λ denotes length as an R-module, coincides with a polynomial $P_{\mathcal{I}}(x) \in \mathbf{Q}[x]$ of degree d for large n. This polynomial is written as

$$P_{\mathcal{I}}(x) = e_0(\mathcal{I})\binom{x+d-1}{d} - e_1(\mathcal{I})\binom{x+d-2}{d-1} + \cdots + (-1)^d e_d(\mathcal{I}),$$

Received by the editors on November 6, 2011, and in revised form on April 25, 2012.

DOI:10.1216/JCA-2012-4-4-577