JOURNAL OF COMMUTATIVE ALGEBRA
Volume 4, Number 4, Winter 2012

RELATIVE COHEN-MACAULAYNESS AND RELATIVE UNMIXEDNESS OF BIGRADED MODULES

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ABSTRACT. In this paper we study the finitely generated bigraded modules over a standard bigraded polynomial ring that are relative Cohen-Macaulay or relatively unmixed with respect to one of the irrelevant bigraded ideals. A generalization of Reisner's criterion for Cohen-Macaulay simplicial complexes is considered.

Introduction. Let $S = K[x_1, \ldots, x_m, y_1, \ldots, y_n]$ be the standard bigraded polynomial ring over a field K. We set $P = (x_1, \ldots, x_m)$ and $Q = (y_1, \ldots, y_n)$. Let M be a finitely generated bigraded S-module. In [**11**] we call M relative Cohen-Macaulay with respect to Q if we have only one non-vanishing local cohomology with respect to Q. In other words, $\operatorname{grade}(Q, M) = \operatorname{cd}(Q, M)$ where $\operatorname{cd}(Q, M)$ denotes the cohomological dimension of M with respect to Q.

In [**11**], it is shown that if M is a finitely generated bigraded Cohen-Macaulay S-module, then M is relative Cohen-Macaulay with respect to P" if and only if "M is relative Cohen–Macaulay with respect to Q." In Section 1, inspired by this result, we raise the following question: if M is relative Cohen-Macaulay with respect to P and Q, is M Cohen-Macaulay? We have an example in dimension 2 which shows that this is not true in general. The question has a positive answer in some special cases.

Next we show M to be relatively unmixed with respect to Q if $\operatorname{cd}(Q, M) = \operatorname{cd}(Q, S/\mathfrak{p})$ for all $\mathfrak{p} \in \operatorname{Ass} M$. We prove that relative Cohen-Macaulay modules with respect to Q are relatively unmixed with respect to Q but the converse is not true in general. The converse is true under some additional assumptions.

2010 AMS *Mathematics subject classification.* Primary 3D45, 16W50, 13C14, 13F55.

The first author was supported in part by a grant from IPM (No. 89130115). The second author was supported in part by a grant from IPM (No. 89130050).
Received by the editors on March 31, 2011, and in revised form on November 17, 2011.

DOI:10.1216/JCA-2012-4-4-551