JOURNAL OF COMMUTATIVE ALGEBRA
Volume 4, Number 4, Winter 2012

IDEALS GENERATED BY ADJACENT 2-MINORS

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ABSTRACT. Ideals generated by adjacent 2-minors are studied. First, the problem when such an ideal is a prime ideal as well as the problem when such an ideal possesses a quadratic Gröbner basis is solved. Second, we describe explicitly a primary decomposition of the radical ideal of an ideal generated by adjacent 2-minors, and challenge the question of classifying all ideals generated by adjacent 2-minors which are radical ideals.

Introduction. Let $X = (x_{ij})_{\substack{i=1,\dots,m\\ j=1,\dots,n}}$ be an $m\times n$-matrix of indeterminates, and let K be an arbitrary field. The ideals of t-minors $I_t(X)$ in $K[X] = K[(x_{ij})_{\substack{i=1,\dots,m\\ j=1,\dots,m}}]$ are well understood. A standard reference for determinantal ideals are the lecture notes [**2**] by Bruns and Vetter. See also [**1**] for a short introduction to this subject. Determinantal ideals and the natural extensions of this class of ideals, including ladder determinantal ideals arise naturally in geometric contexts which partially explains the interest in them. One nice property of these ideals is that they are all Cohen-Macaulay prime ideals.

Motivated by applications to algebraic statistics, one is led to study ideals generated by an *arbitrary* set of 2-minors of X. We refer the interested reader to the article [**3**] of Diaconis, Eisenbud and Sturmfels where the encoding of the statistical problem to commutative algebra is nicely described. In this paper we concentrate on studying ideals generated by adjacent 2-minors, that is, minors of the form $x_{i,j}x_{i+1,j+1} - x_{i+1,j}x_{i,j+1}$. Hoşten and Sullivant [**9**] describe in a very explicit way all the minimal prime ideals for the ideal generated by all adjacent 2-minors of an $m \times n$-matrix. In Theorem 3.3 we succeed

2010 AMS *Mathematics subject classification.* Primary 13P10, 13C13, Secondary 13P25, 62H17.

Keywords and phrases. Binomial ideals, ideals of 2-adjacent minors, contingency tables.
Received by the editors on April 25, 2011, and in revised form on August 30, 2012.

DOI:10.1216/JCA-2012-4-4-525