

ALMOST COHEN-MACAULAY AND ALMOST REGULAR ALGEBRAS VIA ALMOST FLAT EXTENSIONS

MOHSEN ASGHARZADEH AND KAZUMA SHIMOMOTO

ABSTRACT. The present paper deals with various aspects of the notion of almost Cohen-Macaulay property, which was introduced and studied by Roberts, Singh and Srinivas. For example, we prove that, if the local cohomology modules of an algebra T of a certain type over a local Noetherian ring are almost zero, T maps to a big Cohen-Macaulay algebra.

1. Introduction. Let (R, \mathfrak{m}) be a d -dimensional Noetherian local ring with a system of parameters $\underline{x} := x_1, \dots, x_d$. Hochster's *Monomial Conjecture* states that $x_1^t \cdots x_d^t \notin (x_1^{t+1}, \dots, x_d^{t+1})$ for all $t \geq 0$. The Monomial Conjecture is known to hold for all equicharacteristic local rings and for all local rings of dimension at most 3. A recent proof of this conjecture in dimension 3 due to Heitmann has opened a new approach to the study of homological conjectures in mixed characteristic, and this approach is a sample of *Almost Ring Theory*. We direct the reader to [9] for a systematic study of almost ring theory.

Let R^+ denote the integral closure of a domain R in an algebraic closure of the fraction field of R . Using extraordinarily difficult methods, Heitmann [12] recently proved that R^+ is almost Cohen-Macaulay for a complete local domain R of mixed characteristic in dimension 3. Let T be an R -algebra equipped with a value map v (this term together with its normalized version is explained below, but we warn the reader that the value map is defined on algebras that are not necessarily integral domains). We recall from [18] that T is *almost Cohen-Macaulay*, if every fixed element of the local cohomology module $H_{\mathfrak{m}}^i(T)$ is annihilated by elements of arbitrarily small valuations with respect to v for all $i \neq d$, and $T/(\underline{x})T$ is not almost zero for every system of parameters

2010 AMS *Mathematics subject classification.* Primary 13H10, 13D45.

Keywords and phrases. Almost zero module, big Cohen-Macaulay algebra, coherent ring, flat extension, local cohomology module, non-Noetherian ring, rings of finite global dimension.

Received by the editors on July 20, 2011, and in revised form on February 20, 2012.

DOI:10.1216/JCA-2012-4-4-445 Copyright ©2012 Rocky Mountain Mathematics Consortium