GORENSTEIN PROJECTIVE DIMENSION
WITH RESPECT TO
A SEMIDUALIZING MODULE

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Dedicated to the memory of Colleen Killer

ABSTRACT. We introduce and investigate the notion of $G_C$-projective modules over (possibly non-Noetherian) commutative rings, where $C$ is a semidualizing module. This extends Holm and Jørgensen's notion of $C$-Gorenstein projective modules to the non-Noetherian setting and generalizes projective and Gorenstein projective modules within this setting. We then study the resulting modules of finite $G_C$-projective dimension, showing in particular that they admit $G_C$-projective approximations, a generalization of the maximal Cohen-Macaulay approximations of Auslander and Buchweitz. Over a local ring, we provide necessary and sufficient conditions for a $G_C$-approximation to be minimal.

1. Introduction. Over a Noetherian ring $R$, Foxby [9], Golod [10] and Vasconcelos [19] independently initiated the study of semidualizing modules (under different names): a module $C$ is semidualizing if $\text{Hom}_R(C,C) \cong R$ and $\text{Ext}_R^{\geq 1}(C,C) = 0$. Examples include the rank 1 free module and a dualizing (canonical) module, when one exists. Golod [10] used these to define $G_C$-dimension, a refinement of projective dimension, for finitely generated modules. The $G_C$-dimension of a finitely generated $R$-module $M$ is the length of the shortest resolution of $M$ by so-called totally $C$-reflexive modules; see Definition 4.1. Motivated by Enochs and Jenda's extensions in [7] of Auslander and Bridger's $G$-dimension [2], Holm and Jørgensen [12] have extended this notion to arbitrary modules over a Noetherian ring. The current paper provides a unified and generalized treatment of these concepts, in part by removing the Noetherian hypothesis. The tools