JOURNAL OF COMMUTATIVE ALGEBRA
Volume 1, Number 4, Winter 2009

ON MODULES M FOR WHICH $N \cong M$ FOR EVERY SUBMODULE N OF SIZE $|M|$

GREG OMAN

ABSTRACT. Let R be a commutative ring with identity, and let M be an infinite unitary R-module. M is called a *Jónsson module* provided every submodule of M of the same cardinality as M is equal to M. Such modules have been well-studied, most notably by Gilmer and Heinzer ([**3–6**]). We generalize this notion and call M *congruent* provided every submodule of M of the same cardinality as M is isomorphic to M (note that this class of modules contains the class of Jónsson modules). These modules have been completely characterized by Scott in [**10**] when the operator domain is $\mathbf{Z}$. In [**9**], the author extended Scott's classification to modules over a Dedekind domain. In this paper, we study congruent modules over arbitrary commutative rings. We use the theory developed in this paper to prove new results about Jónsson modules as well as characterize several classes of rings.

1. Introduction and general results. In this paper, all rings are assumed commutative with identity and all modules are unitary. We begin by revisiting the definition given in the abstract.

Definition 1. Let M be an infinite module over the ring R. We call M a congruent module if and only if whenever N is a submodule of M of the same cardinality as M, then $N \cong M$.

To initiate the reader and to motivate our study, we introduce some canonical examples of congruent modules.

Example 1. Let F be an infinite field. Then F becomes a module over itself whose submodules are precisely the ideals of F. Since F has only trivial ideals, it is easy to see that F is congruent as a module over itself.

2000 AMS *Mathematics subject classification.* Primary 13C05, 03E10.
Received by the editors on December 28, 2007, and in revised form on January 26, 2008.

DOI:10.1216/JCA-2009-1-4-679