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A CLASSIFICATION OF THE INTEGRALLY CLOSED RINGS OF POLYNOMIALS CONTAINING $\mathbb{Z}[X]$

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ABSTRACT. We study the space of valuation overrings of $\mathbb{Z}[X]$ by ordering them using a constructive process. This is a substantial step toward classifying the integrally closed domains between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$ that are Prüfer, the ones that are Noetherian, and the ones that are PvMDs, to name a few.

1. Introduction. We start with a brief, but substantial, preface. As stated in the abstract, the aim of this paper is to investigate the structure of the integrally closed overrings of $\mathbb{Z}[X]$ by using order-theoretic arguments on the space of the valuation overrings of $\mathbb{Z}_p[X]$ for a prime number p. The technical machinery used to reach this goal is essentially derived from MacLane's paper [19] and all the results that we use are proven in [19] for $R_P[X]$, where R is ANY Dedekind domain with finite residue fields and P is any maximal ideal. Without loss of generality with respect to MacLane's hypothesis, we chose to restrict to overrings of $\mathbb{Z}[X]$ for ease of comprehension (to balance the difficulty of the many technical aspects). But all the results given in the following can be proven exactly in the same way by replacing $\mathbb{Z}$ with any Dedekind domain with finite residue fields and considering prime elements instead of prime numbers.

Let $\mathbb{Z}$ be the ordinary ring of rational integers and let p be a fixed prime number. This paper began as the first step in an attempt to understand the structure of the set of Prüfer overrings of $\mathbb{Z}_p[X]$ by understanding the structure of the collection of valuation overrings of $\mathbb{Z}_p[X]$. The more particular focus was the collection of Prüfer domains

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