Note on Nonlinear Differential Equation of Catalysis¹⁾

By

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1. Mr. Yoshiyuki Suehiro, a chemical engineer, has consulted the author about the solutions of his differential equation of catalysis by spherical tablets,

(1)
$$\begin{cases} (m-1)\left(Da\frac{dy}{dx} - wa\frac{y}{t}\right) = wa, \\ \frac{dwa}{dx} dx = cy\frac{adx}{r}; \end{cases}$$

or for the case m=2, eliminating w in (1), we have

$$(2) \qquad \frac{d}{dx} \left(\frac{Dat}{y+t} \frac{dy}{dx} \right) = \frac{ca}{r} y;$$

or fully written,

(3)
$$\frac{d^2y}{dx^2} + \frac{2}{x} \frac{dy}{dx} - \frac{1}{y+t} \left(\frac{dy}{dx}\right)^2 - n^2 \frac{y(y+t)}{t} = 0,$$

where $n=\sqrt{C/D_T}$. In these equations, x means the distance of any point of the tablet from its centre; y is the concentration of the reacting substance at x; w is the quantity of mass flow of the reacted substance through the spherical surface of radius x in the tablet; a is the total area occupied by the pores on the spherical surface of radius x, and hence proportional to x^2 , while the remainings are chemical constants, positive. Solutions for x > 0, satisfying the conditions $\frac{dy}{dx} = 0$ at x = 0, are required.

2. We may suppose t=1 by writing y instead of ty (also n=1 by writing x instead of nx).

Read before the last autumn meeting of Japanese Mathematical Society held in Kyoto.