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Higher composition products

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0. If W, X, Y, and Z are topological spaces with base point, and $\alpha \in \Pi(W, X)$, $\beta \in \Pi(X, Y)$, $\gamma \in \Pi(Y, Z)$ are homotopy classes of base point respecting maps such that the compositions $\gamma \circ \beta$ and $\beta \circ \alpha$ are zero in $\Pi(X, Z)$ and $\Pi(W, Y)$ respectively, Toda [11] defined the triple product $\{\gamma, \beta, \alpha\} \subseteq \Pi(SW, Z)$. He has since employed it to great advantage in studying the homotopy groups of spheres ([14], [16]). It has often been noted that the triple product bears a formal resemblance to the Massey triple product. In [7], Massey showed how to define longer products analogous to the Massey products, and in [10] Spanier showed how to define longer products analogous to the triple product. It is the object of this paper to give another definition of longer composition products analogous to the triple product, and to explore some of their properties. The advantage of this definition, as will be seen in Sections 5 and 6, is that it enables us to make certain computations in the homotopy groups of spheres. It is, unfortunately, a very cumbersome definition; it is hoped to give a more categorical approach to it in a later paper. These products seem related to those defined by D. M. Kahn (private communication). Their relation to those defined by Spanier is unclear.

1. In this paper all spaces will have the homotopy type of a

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