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A note on stunted lens space

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The purpose of the present note is to show some results on stunted lens spaces analogous to those on stunted projective spaces [5], [4], [3]. Throughout the note a prime p will always be odd.

Let S^{2n+1} be the unit $(2n+1)$-sphere, each point of which is represented by a sequence $(c_0, c_1, \cdots, c_n)$ of complex numbers c_i with $\sum|c_i|^2=1$. $S^1=U(1)$ is a group operating on S^{2n+1} by the formula $c.(c_0, \cdots, c_n)=(c.c_0, \cdots, c.c_n)$. Let p be an odd prime and let Z_p be a subgroup of S^1 generated by $e^{2\pi i/p}$. Then

$$L^n(p)=S^{2n+1}/Z_p \quad \text{and} \quad CP(n)=S^{2n+1}/S^1$$

are the $(2n+1)$-dimensional lens space and the complex projective space of complex n-dimension respectively. Let $\{c_0, \cdots, c_n\}\in L^n(p)$ denote the class of $(c_0, \cdots, c_n)\in S^{2n+1}$. The space $L^k(p)$, $k\leq n$, is naturally imbedded in $L^n(p)$ by identifying $\{c_0, \cdots, c_k\}$ with $\{c_0, \cdots, c_k, 0, \cdots, 0\}$. Denote $L_0^k(p)=\{\{c_0, \cdots, c_k\}\in L^k(p)\mid c_k: \text{ real}, c_k\geq 0\}$. Then $L^k(p)-L_0^k(p)$ and $L_0^k(p)-L^{k-1}(p)$, $k\leq n$, are $(2k+1)$- and $2k$-cells which make $L^n(p)$ a finite CW-complex.

For the class $\alpha\in KO(X)$ of a real s-vector bundle over a finite CW-complex X, X^α will denote the associated Thom complex, i.e., a mapping cone of an $(s-1)$-sphere bundle p: $E\rightarrow X$ associated with α. A cellular decomposition $X=\sum e_i^{n_i}$ of X gives naturally a cellular decomposition $X^\alpha=e^0+\sum e_i^{s+n_i}$ of X^α.

Denote by $\xi(\in K(CP(n)))$ the canonical line bundle (or its class) of $CP(n)$, by $r(\xi)(\in KO(CP(n)))$ the real restriction of ξ.