

A note on theorems of Bott and Samelson

By

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Introduction.

Let there be given an n (≥ 2)-dimensional complete and connected Riemannian manifold M of class C^∞ . Throughout this paper, let a geodesic be parametrized by arc length, unless otherwise stated. A *geodesic loop* is a geodesic segment for which the initial and the final points coincide. Let γ be a geodesic loop parametrized by arc length s ($0 \leq s \leq 2l$) such that $\gamma(0) = \gamma(2l)$, where self-intersections are permitted. γ is said to be *fundamental*, if there is no parameter s such that $\gamma(s) = \gamma(0)$ for $0 < s < 2l$. Throughout this paper, we mean by a *geodesic loop* a fundamental geodesic loop. A *closed geodesic* is by definition a geodesic loop whose initial tangent vector coincides with the final tangent vector. In connection with the study of homological properties of compact irreducible symmetric spaces of rank one, Bott [3]¹⁾ has studied the homological structure of Riemannian manifolds M having the following properties:

- (a) there exists a point p such that all geodesics starting from p are closed,
- (b) all of these closed geodesics passing through the point p are of the same length,
- (c) all of these closed geodesics passing through the point p are simple and of the same index λ .

Making use of the Morse theory of the loop space and the

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1) Numbers in brackets refer to the Bibliography at the end of this paper.