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Branching Markov processes I

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Introduction. Let S be a compact Hausdorff space with a countable open base, S^n the n-fold symmetric product of S, $\boldsymbol{S}=\bigcup_{n=0}^{\infty} S^n$ the topological sum of S^n, where $S^0=\{\partial\}$, ∂ an extra point, and $\hat{\boldsymbol{S}}=\boldsymbol{S}\cup\{\Delta\}$ the one-point compactification of $\boldsymbol{S}$. The purpose of this paper is to investigate a class of semi-groups $\{\boldsymbol{T}_t;\, t\geq 0\}$ of linear operators defined on the space $\boldsymbol{B}(\hat{\boldsymbol{S}})$ of bounded measurable functions on $\hat{\boldsymbol{S}}$ with a special property, which will be called the *branching property*;

$$\boldsymbol{T}_t\hat{f}(\boldsymbol{x})=\widehat{(\boldsymbol{T}_t\hat{f})|_S}(\boldsymbol{x}),\quad \boldsymbol{x}\in\boldsymbol{S},\ \hat{f}\in\boldsymbol{B}(S), \tag{1}$$

where $\wedge$ is a mapping from $\boldsymbol{B}(S)$, the space of bounded measurable functions on S, to $\boldsymbol{B}(\hat{\boldsymbol{S}})$ defined by

$$\hat{f}(\boldsymbol{x})=\begin{cases}\prod_{j=1}^{n} f(x_j), & \text{when } \boldsymbol{x}=[x_1, x_2, \cdots, x_n],\\ 1, & \text{when } \boldsymbol{x}=\partial,\\ 0, & \text{when } \boldsymbol{x}=\Delta.\end{cases} \tag{2}$$

When the semi-group $\boldsymbol{T}_t$ is positivity preserving and contraction, there corresponds a Markov process on $\hat{\boldsymbol{S}}$ with the semi-group by the general theory of Markov processes. We shall call the Markov process a *branching Markov process*. Branching processes are investigated by many authors as a mathematical model for the population

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