J. Math. Kyoto Univ.
8-2 (1968) 215-231

Generalized bilinear relations on open Riemann surfaces[1)]

By

Michio YOSHIDA

(Communicated by Professor Kusunoki, May 2, 1968)

Introduction

Let W be a Riemann surface of infinite genus. A connected subregion Ω of W will be called *normal* if it has positive finite genus and its relative boundary consists of a finite number of mutually disjoint dividing analytic Jordan curves. We stress that a normal subregion is not necessarily a relatively compact region. It may have, besides the relative boundary, the ideal boundary.

In §§2, 3, we introduce two intrinsic conformal invariants $\mu(\bar{\Omega})$ and $M(\Omega)$ for a normal subregion Ω.

Suppose that W is decomposed into a sequence $\{\Omega_k\}_{k=1}^{\infty}$ of normal subregions. Consider a canonical homology basis $\{A_j^k, B_j^k\}_{j=1}^{g_k}$ of Ω_k modulo dividing cycles. Set $D_n = \bigcup_{k=1}^{n} \Omega_k$. The main purpose of this paper is to establish the following evaluations which lead to a generalized bilinear relation:

Suppose $\mu(\bar{\Omega}_k) \leqq \mu$ *(resp.* $M(\Omega_k) \leqq M$*),* $k=1, 2, \cdots$*. Then, we have*

$$|(\sigma, \omega^*) - \sum_{k=1}^{n} \sum_{j=1}^{g_k} \left(\int_{A_j^k} \sigma \int_{B_j^k} \bar{\omega} - \int_{B_j^k} \sigma \int_{A_j^k} \bar{\omega} \right)|$$
$$\leqq (1+\mu) \|\sigma\|_{W-D_n} \|\omega\|_{W-D_n} \ (\mathit{resp.}\ 2M \|\sigma\|_{W-D_n} \|\omega\|_{W-D_n})$$

for all $\sigma, \omega^* \in \Gamma_{h0}(W) \cap \Gamma_{hse}^*(W)$.[2)]

1) This is a revised version of the contents of the talk given at a seminar held at Kyoto University on March 14, 1966.

2) In this paper, differentials are complex in general.