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On the modulus of continuity of sample functions of Gaussian processes

By

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1. Introduction

It is interesting that many important properties of sample functions of $G.p.$[1] hold with probability 0 or 1. For some class of $G.p.$'s the modulus of continuity of sample functions is one of such properties. Let $\{X(s)\,;\, s\in D\}$ be a real valued $G.p.$ with a parameter space D. We shall throughout this paper assume the following.

(A. 1) D is a compact convex subset of N-dimensional Euclidean space containing an open set with the usual Euclidean metric $\|s-t\|^2=\sum_{i=1}^{N}(s_i-t_i)^2$.

(A. 2) $\{X(s)\,;\, s\in D\}$ has the mean $E[X(s)]=0$ and with stationary increment $\sqrt{E[(X(s)-X(t))^2]}=\sigma(\|s-t\|)$, where $\sigma^2(x)$ ($\sigma(x)\not\equiv 0$) is concave near the origin and $\sigma(x)$ is a non-decreasing continuous function that satisfies

$$(1.1)\qquad \int_0^\infty \sigma(e^{-x^2})dx<+\infty\,.$$

This condition guarantees that the $G.p.$ has continuous sample functions by modification due to the theorem of X. Fernique [5]. So we shall assume that the $G.p.$ has continuous sample functions.

Now we are led to introduce the concept of upper class and lower class with respect to the modulus of uniform continuity or local continuity of sample functions.

Let $\varphi(x)$ be a non-increasing continuous function satisfying

1) $G.p.$ means Gaussian process.