On The Existence of Solutions of Stochastic Differential Equations with Boundary Conditions

By

Shintaro NAKAO

(Communicated by Professor Yoshizawa, July 26, 1971)

§0. Introduction.

One-dimensional diffusion processes were studied by Feller, Itô, McKean and Dynkin. On the other hand multi-dimensional diffusion processes have been studied in various points of view. Ventcel' [15] pointed out that under suitable regularity conditions, a diffusion process on a smooth manifold $\bar{D}=D\cup\partial D$ of *n* dimensions with a smooth boundary is determined by the following (A, L, ρ) . Suppose (ψ, U) is a coordinate mapping with the following property,

$$\psi^{1}(x) > 0 \Leftrightarrow x \in D \cap U,$$

$$\psi^{1}(x) = 0 \Leftrightarrow x \in \partial D \cap U.$$

A is an elliptic differential operator of second order which is expressed in the form,

$$Af(x) = \sum_{i,j=1}^{n} a^{ij}(x) D_{ij}f(x) + \sum_{i=1}^{n} b^{i}(x) D_{i}f(x) + c(x)f(x),$$

where $(a^{ij}(x))$ is symmetric and positive semi-definite and $c(x) \leq 0$. L is an operator which maps a smooth function on \overline{D} to a function on ∂D given in the form,