J. Math. Kyoto Univ. (JMKYAZ)
12-1 (1972) 129-140

Polyharmonic classification of Riemannian manifolds

By

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(Communicated by Professor Kusunoki, July 1, 1971)

A *polyharmonic function* is a C^{2n} solution, $n \geq 2$, of the equation

$$\Delta^n u = 0. \tag{1}$$

We sometimes also use the term *n-harmonic* to specify the degree. The object of the present study is a polyharmonic classification of Riemannian manifolds, i.e. the problem of existence of polyharmonic functions with various boundedness properties. We shall show that much of the biharmonic classification theory developed in Nakai-Sario [4], [5], Sario-Wang-Range [9], and Kwon-Sario-Walsh [2], can be generalized to the polyharmonic case. The higher degree brings forth fascinating new versality, as various boundedness conditions can be separately imposed on the functions and the iterates of the Laplacian.

In §1 we introduce the quasipolyharmonic classification of Riemannian manifolds based on the equation $\Delta^n u = 1$, and characterize the corresponding null classes in terms of the harmonic Green's function. Polyharmonic projection and decomposition are the topics of § 2. As an application we find a necessary and sufficient condition for the existence of a solution of the polyharmonic Dirichlet problem. We also briefly discuss the classification theory associated with the class of q-polyharmonic functions.

The work was sponsored by the U. S. Army Research Office-Durham, Grant DA-ARO-D-31-124-71-G20, University of California, Los Angeles.