J. Math. Kyoto Univ. (JMKYAZ)
13-3 (1973) 529-535

Bounded polyharmonic functions and the dimension of the manifold

By

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(Communicated by Professor Kusunoki, November 6, 1972)

Let H^2B be the class of bounded biharmonic nonharmonic functions, i.e., nondegenerate solutions of $\Delta^2 u=0$, with Δ the Laplace-Beltrami operator $d\delta+\delta d$. Consider the punctured space E_α^N: $0<|x|<\infty$, $x=(x^1,\dots,x^N)$ with the metric $ds=|x|^\alpha|dx|$, α a constant. It was shown in Sario-Wang [1] that although E_α^N with $N=2,3$ carries H^2B-functions for infinitely many values of α, it tolerates no H^2B-functions for any α if $N\geq 4$. In the present paper we ask: What can be said about the class H^kB of bounded nondegenerate polyharmonic functions of degree k, that is, solutions of $\Delta^k u=0$? The answer turns out to be rewarding and puts the biharmonic case in proper perspective: *There exist no H^kB-functions on E_α^N for any α if $N\geq 2k$.*

For $N<2k$ there are infinitely many α for which these functions do exist, and for these α the generators of the space H^kB are surface spherical harmonics. In particular, this is true of H^2B-functions on Euclidean 2- and 3-spaces, as was recently shown in Sario-Wang [2].

If $H^kB\neq\emptyset$ on a given E_α^N, is the same true of H^hB for any $h>k$? We shall show that, while this is so for every N if the metric of E_α^N is Euclidean, there are values of (N,α) for which it does not hold.

AMS 1970 subject classification 31B30.

The work was sponsored by the U. S. Army Research Office-Durham, Grant DA-ARO-D-31-124-71-G181, University of California, Los Angeles.