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On amenable transformation semigroups II

By

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§ 0. Introduction

This paper is the continuation of my preceding paper [20] which is referred as Part I. So we shall employ the same notations and definitions as in Part I. Let S be a semigroup, $\boldsymbol{X}=(S, X)$ a transformation semigroup (denoted by τ-semigroup briefly) and $\mathfrak{A}$ any $\boldsymbol{X}$-linear space. In Part I we have studied several necessary and sufficient conditions for $\mathfrak{A}$ to be amenable, extremely amenable or quasi-extremely amenable, which are stated in terms of some intrinsic properties of $\mathfrak{A}$, $\mathfrak{A}^*$ or $\boldsymbol{X}$. While the purpose in this paper is to show that the amenability or extreme amenability of $\mathfrak{A}$ can be characterized by means of certain properties concerning actions of $\boldsymbol{X}$ on various objects, and to apply these characterizations to the case of left amenability of any $\boldsymbol{S}_l$-linear spaces. The above characterizations are certain generalizations of the results on left amenability of $B(S)$ or of some $\boldsymbol{S}_l$-linear spaces investigated by a number of authors, e.g., T. Mitchell [14], B. E. Johnson [9], E. E. Granirer [8] and so on.

All the discussions in this paper do not depend on the topological structure of underlying τ-semigroup and do not require any additional conditions on $\mathfrak{A}$ except that $\mathfrak{A}$ is an $\boldsymbol{X}$-linear space or $\boldsymbol{X}$-algebra. The amenability associated with topological τ-semigroups will be studied in the author's paper [21], in which we shall apply the present results to the cases of function spaces of special kinds.