J. Math. Kyoto Univ. (JMKYAZ)
17-1 (1977) 69-90

On uniqueness of analytic solution for first order partial differential equations with degenerate principal symbols II

By

Akira NAKAOKA

(Communicated by prof. Mizohata, March 17, 1976)

1. Introduction

In the previous paper [5], the author treated the following equation;

$$a(x, y)\partial u/\partial x+b(x, y)\partial u/\partial y=c(x, y)u, \tag{1.1}$$

where $a(x, y)$, $b(x, y)$ and $c(x, y)$ are holomorphic in a neighborhood of the origin of $\boldsymbol{C}^2$, and discussed the uniquness of the solution which is analytic in a neighborhood of the origin of $\boldsymbol{C}^2$. There, under the following hypothesis (H);

(H) $\partial^{p+q}c(0,0)/\partial x^p\partial y^q=0$ $(p+q=0, 1, \cdots, m-1)$ *for some natural number* m, *and for this* m *there exist* p *and* q *with* $p+q=m$ *such that* $\partial^m c(0,0)/\partial x^p\partial y^q\neq 0$. *Moreover, it holds* $\partial^{p+q}a(0,0)/\partial x^p\partial y^q=\partial^{p+q}b(0,0)/\partial x^p\partial y^q=0$,

we obtained.

Theorem. 1. *Let* $A(x, y)$, $B(x, y)$ *and* $C(x, y)$ *denote the homogeneous parts of degree* $m+1$ *of* $a(x, y)$, $b(x, y)$ *and the homogeneous part of degree* m *of* $c(x, y)$ *respectively, and set* $A(x, y)=\omega(x, y)\alpha(x, y)$, $B(x, y)=\omega(x, y)\beta(x, y)$ *and* $C(x, y)=\omega(x, y)\gamma(x, y)$. *Then if the equation*

$$x\beta(x, y)-y\alpha(x, y)=0,\ \ \gamma(x, y)=1 \tag{1.2}$$

has no solution, the solution of (1.1) *which is analytic in a neighborhood is only zero.*

The main aim of this paper is to extend the result of Theorem. 1 into the case of general several variables. Hence our concerning equation is as follows;

$$\sum_{j=1}^{n+1} a_j(x_1, \cdots, x_{n+1})\partial u/\partial x_j=\omega(x_1, \cdots, x_{n+1})u, \tag{1.3}$$

where $a_j(x_1, \cdots, x_{n+1})$ and $\omega(x_1, \cdots, x_{n+1})$ are all holomorphic in a neighborhood of the origin of $\boldsymbol{C}^{n+1}$. Here, corresponding to (H), we assume

$(H.1)$ $D^\alpha\omega(0, \cdots, 0)=0$ *when* $|\alpha|\leqq m-1$ *for some natural number* m