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Lie subalgebras of finite codimension in the restricted Poisson algebra

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Introduction.

Let (M, ω) be a symplectic manifold which is defined by a symplectic form ω. It is well-known that the symplectic Lie algebra L on (M, ω) has the minimum transitive ideal L^* [1]. Let $\mathscr{D}(M)$ be the space of C^∞-functions with compact support. We define a subspace $\mathscr{N}$ of $\mathscr{D}(M)$ by $\mathscr{N}=\{u\in\mathscr{D}(M); \int_M u\omega^n =0\}$, where ω^n is the volume element defined by the symplectic form ω, i.e., $\omega^n=\overbrace{\omega\wedge\cdots\wedge\omega}^{n}$ $(n=1/2\dim M)$. Now L^* is explicitly defined by $L^*=\{X\in L; i(X)\omega=du, u\in\mathscr{N}\}$.

It is well-known that $\mathscr{N}$ gives rise to a Lie algebra with respect to the Poisson bracket and that it is isomorphic to L^* as Lie algebras. We will call this algebre $\mathscr{N}$ the restricted Poisson algebra.

In the present paper, we prove the following fact.

Let $\mathscr{N}$ be the restricted Poisson algebra on (M, ω), and let $\{F_i\}_{1\leq i\leq m}$ be a set of linearly independent linear functionals on $\mathscr{N}$ which are continuous with respect to the canonical topology on $\mathscr{N}$. If $\mathscr{K}=\bigcap_{i=1}^{m} \operatorname{Ker} F_i$ is a Lie subalgebra (which is necessarily of finite codimension) of $\mathscr{N}$, then $A=\bigcup_{i=1}^{m} \operatorname{supp} F_i$ is a finite subset of M and

$$\mathscr{K}\subset\bigcap_{a\in A}\{u\in\mathscr{N}; (du)_a=0\}.$$

In considering the problem above, we have been motivated by a theorem of W. D. Curtis and F. R. Miller [3]. Indeed let M be a C^∞-manifold and $\Gamma_c(TM)$ the Lie algebra of all C^∞-vector fields with compact support on M. Then they obtained an analogous result as ours with respect to a finite set of linear functionals on $\Gamma_c(TM)$, which defines a Lie subalgebra of $\Gamma_c(TM)$.

Let L_0 be the ideal consisting of all vector fields of L which have compact support. This ideal L_0 corresponds to the Lie algebra $\Gamma_c(TM)$ studied by Curtis and Miller, but L_0 has, in general, a Lie subalgebra of finite codimension which is transitive on M. Therefore we consider $\mathscr{N}$ or equivalently L^* instead of L_0.

We now proceed to the description of each section. In § 1, the precise