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Convergence of solutions of one-dimensional semilinear parabolic equations

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Introduction

The idea of ω-limiting sets borrowed from the theory of dynamical systems has been employed in many authors' works including [1] and [5] to investigate the asymptotic properties of solutions of autonomous parabolic initial-boundary value problems. In the case of parabolic systems, it is well-known that the ω-limiting set of a solution often contains plural elements (in fact infinitely many elements). But, to the best of our knowledge, it has not yet been made clear in the case of single equations whether there exists such a solution as has plural ω-limiting points. The present paper forms part of the answer to this question. That is, we show that the ω-limiting set of any solution contains at most one element providing that the space dimension is one. This result leads to the conclusion that in the case of single one-dimensional equations any solution that neither blows up in a finite time nor grows up as $t\to\infty$ should converge to some equilibrium solution as t tends to infinity.

§1. Notation and Theorems

Let us consider initial-boundary value problems of the form

$$u_t=\{a(x)u_x\}_x+f(x, u), \qquad (0<x<L,\ 0<t<s), \tag{1.1a}$$

$$u(x, 0)=u_0(x), \qquad (0\leqq x\leqq L), \tag{1.1b}$$

$$\alpha_0 u(0, t)-(1-\alpha_0)u_x(0, t)=\beta_0, \qquad (0<t<s), \tag{1.1c$_0$}$$

$$\alpha_1 u(L, t)+(1-\alpha_1)u_x(L, t)=\beta_1, \qquad (0<t<s), \tag{1.1c$_1$}$$

where the coefficient $a(x)$ is positive and sufficiently smooth in the compact interval $[0, L]$ and α_i, β_i $(i=0, 1)$ are real constants satisfying $0\leqq\alpha_i\leqq 1$ $(i=0, 1)$. The nonlinear term f is also assumed to be a smooth function, say of class C^2, mapping $[0, L]\times\mathbf{R}$ into $\mathbf{R}$. We choose initial data from the space $C[0, L]$ and consider real-valued classical solutions. A function $v\in C^1[0, L]\cap C^2(0, L)$ is said to be an