J. Math. Kyoto Univ. (JMKYAZ)
20-3 (1980) 395-416

Infinite dimensional stochastic differential equations and their applications

By

Tokuzo SHIGA and Akinobu SHIMIZU

(Communicated by Prof. S. Watanabe, March 27, 1979)

§ 1. Introduction

The central aim of this paper is to construct some classes of infinite dimensional stochastic processes related to population genetics or statistical mechanics by making use of infinite dimensional stochastic differential equations.

To begin with, we will explain an example related to population genetics. Let S be a countable set which we consider as the set of colonies. Suppose that there are two alleles A and a in each colony and that the change of gene frequencies is caused by random sampling, mutation, selection and migration. As is well-known in population genetics ([1], [6]), when we ignore the migration effect, the frequency $x_i(t)$ of A-genes in the i-th colony at time t may be considered a path of 1-dimensional diffusion process on the interval [0, 1] determined by the diffusion coefficient

$$\frac{1}{4N}x_i(1-x_i)$$

and the drift coefficient

$$v-(u+v)x_i+s\,x_i(1-x_i)$$

for each $i\in S$. Hence the frequency $x_i(t)$ satisfies the following 1-dimensional stochastic differential equation

$$dx_i(t)=\sqrt{\frac{1}{2N}x_i(1-x_i)}dB_i(t)+(v-(u+v)x_i+s\,x_i(1-x_i))dt$$

for each $i\in S$, where $\{B_i(t)\}_{i\in S}$ is an independent system of 1-dimens-