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Notes on the Riemann-Roch theorem on open Riemann surfaces

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Introduction

Riemann-Roch theorem, one of the most important theorems in the classical theory of Riemann surfaces, was at first extended to open Riemann surfaces by Y. Kusunoki [4], and afterwards generalized along his method by H. Mizumoto [7], M. Yoshida [12] and M. Shiba [9]. Comparing these generalizations, however, they can be classified superficially into two types, namely the generalization by Mizumoto and those of Yoshida and Shiba have somewhat different forms, where Shiba's result is clearly an extension of Yoshida's one. Whereas the relationship between the Mizumoto's result and Yoshida's one was not known, and so we intend in this paper to discuss about this relationship.

In this paper, we recall in § 1 the notion of Yamaguchi's regular operators and some related results (Yamaguchi [11]), and next in § 2 and § 3, we consider the convergence of the sequence of the certain harmonic functions by using the regular operator's method (Cf. Theorem 1). Finally, in § 4, by applying the results in § 2 and § 3, we show that the Yoshida's theorem can be regarded as an extension of the Mizumoto's one (Cf. Theorem 2). As for the notations and the terminologies concerning the differentials in this paper, we shall use those in Ahlfors and Sario [1] without repetitions, though we restrict ourselves to real differentials.

§ 1. Regular operator

Let R be an open Riemann surface, W an end towards the Alexandroff's ideal boundary Δ of R (namely, the complement of W is the closure of a regular region of R) and $\{R_n\}$ a regular exhaustion of R. Denote $W\cup\partial W$ by $\overline{W}$ and set

$HD(R)=a$ Banach space of harmonic Dirichlet functions on R with respect to the norm $|||u|||=\|du\|+|u(a_o)|$, where $u\in HD(R)$, $\|du\|$ the Dirichlet norm on R of du and a_o is a fixed point on R,

$D_o(R)=$the set of all Dirichlet potentials on R,

$X=a$ subspace of $HD(R)$,

$C^\omega(\partial W)=\{f: f$ is a real analytic function on the relative boundary ∂W of $W\}$,

$H(\overline{W})=\{$restriction to $\overline{W}$ of a harmonic function on an open set containing $\overline{W}\}$.