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A uniqueness theorem for functions holomorphic in a half-plane

By

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1. Introduction

In [1, p. 75], R. P. Boas states and proves the following uniqueness theorem.

Theorem 1. *If $f(z)$ is an entire function of exponential type and if for some θ, $|f(re^{i\theta})| \leq e^{-A(r)r}$, where $\lim_{r\to\infty} A(r) = \infty$, then $f(z) = 0$ identically.*

This theorem shows that if the function $f(z)$ tends to zero sufficiently rapidly along some ray $\arg z = \theta$, then it must be identically zero. Instead of a ray, in [4, Theorem 1], we have extended the same result to an arbitrary curve having a limit point at infinity. Moreover, the same type of uniqueness theorems can also be considered in a half-plane instead of the entire plane, see [3, 6]. In this connection, instead of a curve, we can also regard only a sequence of Jordan arcs tending to the point at infinity, see [5]. The shape of Jordan arcs can behave in different ways. The one considered in [5] looks like arc, and the other which we are going to discuss looks like radial. In this case, our results are somewhat equivalent to those of V. I. Gavrilov [2] and D. C. Rung [8] in the unit disk.

To introduce our results, let us begin with the following two definitions.

Definition 1. Let $H = \{z: \operatorname{Re} z > 0\}$ be the right halfplane and let $\{\gamma_n\}$ be a sequence of disjoint Jordan arcs in H. Denote α_n the angle subtended by γ_n at the origin and set

$$l_n = \min_{z\in\gamma_n} |z|, \ L_n = \max_{z\in\gamma_n} |z|, \ \theta_n = \min_{z\in\gamma_n} \arg z.$$

We call $\{\gamma_n\}$ a radial-like sequence if it tends to infinity nontangentially and satisfies the following two conditions

(i) $$\lim_{n\to\infty} l_n = \lim_{n\to\infty} L_n = \infty, \quad \text{and}$$

(ii) $$0 < \varliminf_{n\to\infty} l_n/L_n = \varlimsup_{n\to\infty} l_n/L_n < 1.$$

Definition 2. Let $\{\gamma_n\}$ be a radial-like sequence and let α_0 be a fixed angle sati-